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Training Schrödinger's cat: quantum optimal control

Strategic report on current status, visions and goals for research in Europe

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Abstract. [zeroth draft] It is control that turns scientific knowledge into useful technology: in physics and engineering it provides a systematic way for driving a system from a given initial state into a desired target state with minimized expenditure of energy and resources – as famously applied in the Apollo programme. As one of the cornerstones for enabling quantum technologies, optimal quantum control keeps evolving and expanding into areas as diverse as quantum-enhanced sensing, manipulation of single spins, photons, or atoms, optical spectroscopy, photochemistry, magnetic resonance (spectroscopy as well as medical imaging), quantum information processing and quantum simulation. — Here state-of-the-art quantum control techniques are reviewed and put into perspective by a consortium uniting expertise in optimal control theory and applications to spectroscopy, imaging, quantum dynamics of closed and open systems. We address key challenges and sketch a roadmap to future developments.

PACS. please select by commenting, or suggest more/other: – 02.30.Yy Control theory in mathematical physics – 03.67.Ac Quantum algorithms, protocols, and simulations – 03.67.Lx Quantum computation architectures and implementations – 03.67.Pp Quantum error correction and other methods for protection against decoherence – 07.57.-c Spectroscopy in atomic and molecular physics – 61.05.Qr Nuclear Magnetic Resonance (NMR) in structure determination – 87.61.-c Magnetic Resonance Imaging (MRI) in medical physics – 87.64.kh Electron Spin Resonance (ESR) in biophysics – 89.20.Bb Technological research and development

Foreword

The authors of the present paper represent the QUAINT consortium, a European Coordination Action on Optimal Control of Quantum Systems, funded by the European Commission Framework Programme 7, Future Emerging Technologies FET-OPEN programme and the Virtual Facility for Quantum Control (VF-QC), uniting expertise in optimal control theory and applications to quantum systems both in existing and widely used areas such as spectroscopy and imaging and in emerging quantum technologies such as quantum information processing, quantum communication, quantum simulation and quantum sensing. Challenges to quantum control have been gathered by a broad poll of leading researchers across the communities of general and mathematical control theory, atomic and molecular physics, electron an nuclear magnetic resonance spectroscopy and as medical imaging, quantum information, communication and simulation. 144 experts of these fields have provided feedback and specific input on the state of the art as well as mid-term and long-term goals. These have been summarized in this document, which can be viewed as a perspectives paper, providing a roadmap for the future development of quantum control. As the current document is far from complete and there are many additional areas of quantum control applications, such as spintronics, nano-optomechanical technologies etc., this roadmap is designed as a living document that is available at at the homepage of the VI-QC web page, where additional aspects as well as new developments and ideas will be included.

1 Introduction

It is control that turns scientific knowledge into technology. The general goal of quantum control is to actively manipulate dynamical processes at the atomic or molecular scale, typically by means of external electromagnetic fields or forces. The objective of quantum optimal control is to devise and implement control schemes, shapes of pulses of external fields and sequences of such pulses, that reach a given task in a quantum system in the best way possible. Quantum control builds on a variety of converging theoretical and technological advances from the field of mathematical control theory, numerical mathematics to better electronic devices such as arbitary-waveform generators with sub nanosecond time resolution and stronger magnetic fields.

The challenge to manipulate nature at the quantum level offers a huge potential for current and future applications and for society. Quantum systems and processes cover a wide range from atomic and molecular physics, chemistry, materials (semiconductors, superconductors and novel materials) to biosystems and medicine. Useful applications range from magnetic resonance imaging and spectroscopy to the precise control of chemical reactions to emerging second generation quantum technologies. Quantum optimal control is part of the effort to engineer quantum technologies from the bottom up and many striking examples of surprising and non-intuitive but extremely efficient and robust quantum control techniques have been discovered in recent years. Examples of important current applications are the precise measurement of magnetic fields with nanometer-scale resolution using NV centers in diamond, state engineering of Bose-Einstein condensates and high-fidelity quantum gates in super-conducting quantum processors. Similar to the first generation of quantum-based technology that brought the semiconductor transistor, the laser, magnetic resonance imaging and spectroscopy in the last century, also the currently developing second generation of quantum technology based on superposition, entanglement and many body quantum systems are expected to expand on the potential for new disruptive technologies – from spintronic devices, quantum metrology, computing technology, to elucidating chemical reaction dynamics, material properties to biophysics.

While the precise way to manipulate the behavior of these systems may differ – from ultrafast laser control to radiowaves in nuclear magnetic resonance, the control, identification and system design problems encountered share commonalities, while at the same time being quite distinct from classical control problems. This requires bringing together researchers from different application areas to forge a community, create a common language and identify common challenges. The further development of this field of research offers many beneficial effects for todays and tomorrows society, related to health through faster, better, safer diagnostics and treatment, secure communication in a digital world, highly accurate navigation systems, more efficient and clean harvesting of solar power, the search for resources, efficient energy storage and transportation, quantum machines and precision sensing and monitoring of the environment.

The paper is organized as follows: Section 2 is focussed on mathematical optimal control theory, which is followed by a description of the state of the art and perspectives of quantum control applications in atomic and molecular physics (section 3), magnetic resonance (section 4) and quantum information and communication (section 5).

2 General aspects and mathematics of optimal control

In recent years, some of the advances in quantum control have emerged through the introduction of appropriate and powerful tools coming from mathematical control theory [175, 141, 116, 37, 210, 127, 53, 57] These developments have been recognized as an essential requirement for the future application of quantum technologies. However, many specific theoretical and technical difficulties arise when quantum systems are considered. Due to the unique features of quantum dynamics such as entanglement, the application of control theory has proved to be a non trivial task and has led to the foundation of a new research domain, the so-called *mathematical quantum control theory*. In the last ten years, quantum control has motivated the study of new mathematical objects, which leads to the introduction of new objects in the theory of control systems and it is expected that this will also be true in the future.

The first step in the analysis of the control of a quantum system is to study its controllability, which has a fundamental importance since it indicates the extent to which a quantum system can be manipulated and brought to a desired target state. When the controllability of a system is established, it remains to design the control field. Historically, the first attempts were open-loop control strategies for which the external field was known either analytically or numerically. In this framework, optimal control theory has been successfully developed and applied since the eighties to become nowadays a standard tool both in chemistry and in physics. Different numerical algorithms are currently widely used and able to account for experimental constraints or robustness issues. In spite of their recent success, such open-loop control approaches have limitations due to their intrinsic nature where no feedback from the system dynamics is used to correct the action of the field.

Although closed-loop control methods are standard tools in control theory of macroscopic systems, they raise many difficult questions and subtleties at the quantum level. One reason is related to quantum measurements which unavoidably affect the state of the system. The application of closed-loop strategies has led to an acceleration of the development of other research domains extending from quantum filtering theory to reservoir engineering.

This paragraph is aimed at giving a general overview of the current development of mathematical control theory in the different directions mentioned above. In each case, some short and long-term perspectives will be detailed. We also refer the interested reader to some recent tutorial reviews about the mathematical aspects of quantum control theory [31, 71, 5].

2.1 Controllability and simulability

2.1.1 State of the art

Controllability is a fundamental issue determining whether a quantum system can be brought from any given initial state to any desired target state (where closed systems preserve the eigenvalue spectrum). Taking results from classical linear control systems [116, 210] to bilinear systems with a non-switchable drift term, a rigorous Lie-framework was developed [114, 218, 33, 34, 113, 78]. Based on this work, controllability results are by now well established for closed quantum systems with finite dimensions [4, 57, 66, 137]. Note that only the standard bilinear situation (as in a controlled Schrödinger equation) will be discussed here. Different notions of controllability have been introduced for pure states, mixed states or evolution operator dynamics [198]. The main controllability test is based on the rank of the dynamical Lie algebra, which is generated by the drift and the different control Hamiltonians. characterizing the control system. The difficulty of using the rank condition in large systems led to a geometric approach based on graph theory [6] or a complete set of symmetry criteria for controllability [251]. In infinite-dimensional systems, the mathematics is much more intricate and only few results exist for systems with a discrete spectrum. The first one gives a general obstruction property to exact controllability [229, 157] recently completed by some positive results about exact [18, 19] or approximate controllability based on Galerkin techniques [47]. In another approach, symmetry methods were taken to assess controllability in the infinite-dimensional Jaynes-Cummings model of several two-level atoms coupled to a field [121]. In open Markovian quantum systems, the control field usually cannot fully compensate dissipation effect as rigorously shown in [7]. These works were recently generalized to a complete Lie-semigroup picture [67, 167]. Moreover, simultaneous controllability concerns the control of a continuum of finite dimensional quantum systems by only few control fields. Some mathematical results have been obtained in this direction [19, 144, 143]. This analysis is interesting for designing control fields which are robust to experimental imperfections [230, 128]. This question is crucial in Nuclear Magnetic Resonance for high power machines. Finally, even if a system (A) is not fully controllable, its controlled dynamics may still suffice to generate a set of desired target effective interaction Hamiltonians as brought about by another quantum dynamical system (B). This is pertinent in finite-dimensional quantum simulation. Adapting the tools from control theory, one readily sees that system A can simulate system B if the system algebra of A encompasses that of system B. A recent generalisation of the results in [251] now provides a complete set of symmetry criteria together with an algorithm in order to decide simulability on system-algebraic grounds [253].

2.1.2 Vision and perspectives

A main challenge in quantum controllability is the rigorous understanding of infinite-dimensional Markovian open systems with a discrete spectrum. Extending the standard techniques (Lie-Galerkin) to non-unitary evolutions is non-trivial. With the final goal being sufficient conditions for approximate controllability of open systems analogous to closed systems [47], a mid-term perspective is a well-posed mathematical control problem with sound regularity analysis. Similarly exact or approximate controllability of the Schrödinger equation with mixed or continuous spectrum is still open. Beyond controllability, a precise description of the reachable set (in the open case) and upper bounds on minimal time to reach target states are largely unkown. Also a universal estimate time to control finite dimensional quantum systems (with necessary drift) is still open.

Beyond some controllability results in the Markovian case [7, 67], their extension to general non-Markovian systems is still lacking. Reachability is expected to be larger for non-Markovian dynamics by information back flow environment to system. In an explorative study, non-Markovian *map synthesis* was shown to be indeed stronger than its restriction to the Markovian case (as anticipated in [146]), whereas surprisingly for the simpler problem of state transfer this is not the case [21]. Yet this is but a first step to understanding open quantum systems. From a control point of view, it is expected that the beneficial part of the bath lies in its positive non-Markovian effects, while the detrimental part is within purely Markovian dynamics that cannot be remedied—an aspect important not only for controllability but also for dissipative state engineering and quantum memories.

2.2 Open-loop analytical control strategies

2.2.1 State of the art

Different approaches have been proposed for deriving analytical control fields. First, local control determines the control field from instantaneous dynamical properties of the system by ensuring monotonic increase or decrease of a performance index [80, 215]. Under well-established conditions [238], the system converges asymptotically towards the target. Local controls are reminiscent of the closed-loop Lyapunov method in stabilization. Due to the inherent difficulty of a quantum-measurement process, this approach has been transformed into an open-loop control law in the quantum world. Secondly, adiabatic techniques [22, 236, 101, 223] are usually achieved by a series of intense pulses which can be frequency chirped according to the structure of the energy levels and sufficiently slow so as to fulfill adiabatic conditions. Such processes are robust to small variations of laser or system parameters and thus well-suited to open-loop control. Note the recent controllability results obtained for systems with an infinite-dimensional Hilbert space by using adiabatic arguments [28]. A main drawbacks of adiabatic control is the total time and energy. To improve this aspect, efforts were made to develop a shortcut to adiabaticity [225], an approach that can be viewed as an inverse engineering technique based on Lewis-Riesenfeld invariants as recently applied to a variety of quantum systems [189, 48, 56, 11]. Also connecting the Lewis-Riesenfeld invariants in the approach of shortcuts to adiabaticity with the formal framework of the Pontryagin Maximum Principle of optimal control theory (PMP, see below) would be illuminating.

2.2.2 Vision and perspectives

An important challenge is the extension of the standard adiabatic and shortcut techniques to open quantum systems. Beyond some preliminary results [112, 193], a lot of work remains to be done. One open question is to generalize Lewis-Riesenfeld invariants to such dynamics. Recent results showed that analytic solutions for an ensemble of quantum systems is not out of reach [56]. The basic idea consists in selecting among a family of exact solutions of the dynamics the ones that are effectively robust to some extend to variations of the system parameters.

2.3 Open-loop optimal control strategies

2.3.1 State of the art

Optimal control theory can be viewed as a generalization of the calculus of variations for problems with dynamical constraints. Its modern version was born with Pontryagin's Maximum Principle (PMP) in the late 1950's [175]. Its development was boostered by using Kalman filters [117, 116] in the Apollo programme, but it is now a key tool in many applications including quantum mechanics. Solving an optimal control problem means finding a control law (aka pulse sequence) such that the corresponding trajectory satisfies given boundary conditions, an equation of motion and minimizes a cost criterion (e.g., energy or duration of the control). Usually, first one goes for extremal trajectories solving a generalized Hamiltonian system subject to the maximization condition of the PMP. Secondly, one selects among the extremals those which minimize cost. Although looking straightforward, the practical use of the PMP is far from being trivial and control problems require geometric analysis and numerical methods.

The geometric approach allows a complete geometric understanding of the control problem from which one can deduce the structure of the optimal solution, a proof of global optimality, and physical limits such as minimal time to reach the target. Such results can be determined essentially analytically (or at least with high numerical precision). Yet, geometric control is restricted to low-dimensional problems. In recent years, stronger mathematical tools [3, 23, 113, 27] solved questions of increasing difficulty including fundamental control problems for closed [124, 123, 26, 58, 57, 25, 87] and open quantum systems [24, 140]. Extending methods of geometric control to infinite dimensions [47] was unexpectedly successful.

Numerical optimal control algorithms comprise (i) gradient ascent algorithms [125] extended by second-order Newton or quasi-Newton methods [64, 150] and (ii) Krotov-type methods [134, 151, 183]. They are often easy to apply and to adapt to experimental settings. Several modifications have been brought forward to account for experimental imperfections [243] and to introduce some robustness issues in the optimal solution. In order to cope with dimensionality in many-body quantum systems, gradient-free approaches have been developed such as CRAB [72, 44] interfacing tensor compression of t-DMRG with parameter optimization. Likewise, in the high-temperature regime of ensemble NMR spectroscopy, the SPINACH package [105] uses state-propagation with interactions limited to pertinent short-range ones (thus leading to efficient truncation of the underlying spin Lie algebra) for simulating unprecedentedly large spins systems as entire biomolecules [194] with striking precision.

Standard mathematical results in quantum control show how to design optimal control fields and describe under which conditions they exist. However, such works do not investigate the complexity inherent to the search of optimal control. This complexity is related to the description of the control landscape which specifies the control objective as a function of the control variables. Different results have been established recently about this problem [46, 63, 173, 179]. One of the main findings is the proof that no local trap exist in closed and in some specific open quantum systems [31]. Works remain to be done in this direction to account for the effect of field constraints on the quantum control landscapes or to be able to evaluate the robustness of optimal solutions with respect to noise or experimental imperfections.

2.3.2 Vision and perspectives

A promising direction for numerical optimal control theory lies in open quantum systems, as most systems of interest interact with the environment to non-negligible extent. To this end, system identification has to include dissipative parameters. A first step to combine coherent control with implicit learning about the system parameters is the ADHOC technique [75].

Decoherence in open quantum systems can also be used as a resource in what has become known as dissipative state engineering [38, 234, 133] and likewise, it can be incorporated as additional control parameter [21, 172]. When added to coherent controls, time-dependent Markovian noise (amplitude-damping) enables the control system to transform any initial into any desired target stat [21]. It can be easily integrated in toolboxes like DYNAMO [150], yet it is still unexplored how to implement them in realistic experiments.

It is well-known that decoherence in open quantum systems can create difficulties for coherent quantum control. However, some features of open systems can be advantageous and can be used in the prospective incoherent quantum control [21, 172], for which an active manipulation of the non-unitary part of the dynamics is possible. A rigorous mathematical description of these control processes will be interesting and a first step towards their experimental developments.

A complete and rigorous understanding of the quantum control landscapes remains a crucial challenge. Even if a lot of quantum control problems have control landscapes which are well-suited to local optimization approach, it is not the case in all the situations. In the latter case, global optimization techniques are still numerically expensive and generally do not perform very well. Work remains to be done on such examples to understand the best way to design the optimal control field.

Another challenge for numerical algorithms is stochastic dissipative dynamics. Stochastic methods are potentially useful for systems interacting with environment or subjected to measurements in the sense of being governed by a stochastic process. open question is to find the most efficient way to control such systems by such they are robust with respect to the stochastic parameter.

Generally, constraints can be designed in open-loop control techniques to account for experimental imperfections or limitations. This makes optimal control theory more useful for concrete experiments thus bridging the gap between theory and experiments. In spite of recent progress, a systematic and efficient algorithm is still lacking. As pioneered by [75], another option combines open-loop optimal algorithms to closed-loop control techniques in order to reduce their respective drawbacks. Using these two tools in cooperatively will provide flexible quantum control.

A crucial requirement is still to improve the computational speed and accuracy of the algorithms. This will be an important prerequisite to attack control problems of increasing complexities and close to experimental descriptions.

A final objective is to get an explicit connection between the mathematical control techniques and numerics in view of a general formalism permitting to describe different control applications and to support them by rigorous mathematical results on time optimality, reachability as listed above.

2.4 Quantum feedback control theory

2.4.1 State of the art

A fundamental aspect of the current development of quantum control is based on closed-loop strategies [71] coping with unpredictable disturbances. This approach is widely used in classical control theory, where information from the state of the system is fed back to the controller to correct the field in action. However the perturbing effect of quantum measurements precludes direct application of classical concepts and has led to new techniques tailored to quantum dynamics [245, 69]. Besides also closed-loop learning control is used in quantum control, where each cycle of the loop is entirely realized and a new sample is considered for each cycle. Different learning algorithms such as the genetic ones have been developed to design the new control field after each fundamental iteration [31].

Controlling a quantum system by feedback can be made in two different ways [249], namely the *measurement-based feedback* approach [244] and the coherent feedback method [147]. As in classical control theory, the first option is based on a measurement process and a real-time manipulation of the system through the measurement result. The measurement feedback approach built on quantum filtering techniques [29] is nowadays mathematically well described [245, 30]. Quantum filtering consists in generating an estimate of the state of the system on the basis of the measurements performed on it. The efficiency of *measurement-based feedback* has been shown and discussed in a series of works [51, 52], which renders the real-time control of quantum systems a goal that can be reached experimentally [195, 126]. In the second option, no measurement is used and the quantum system is directly connected to a quantum controller. The experimental feasibility of coherent feedback control leads recently to an impressive amount of works on the subject [110, 95, 149, 108].

2.4.2 Vision and perspectives

It the specific quantum nature of systems that in general makes them affected by measurements. Thus in the quantum domain, finding an experiment-class adapted balance between open-loop control techniques and closed-loop feedback counterparts will be one of the major challenges. Moreover, since measurement-based feedback is restricted to the processing time of the classical components, which have the slowest time scale, exploiting the limits of *coherent feedback control* is useful in view of future technologies. An advantage of coherent feedback over its measurement-based counterpart is the reduced noise produced by the control process since there is no disturbance from a system measurement. Feedback control methods promise particular robustness and flexibility, but some important questions related to the quantum nature of the dynamics remains to be solved to reach such objectives. In particular, there is not a general theory showing that quantum controllers perform better than their classical counterparts, and if so, in which cases and in which experimental conditions. Many other questions are still opened in quantum feedback control. They extend from a general theory about the role of weak measurements in the control of quantum systems to feedback control of non-Markovian dynamics and the influence of model uncertainties on the feedback control.

Most of the developments of quantum feedback control have been made in the context of quantum optics. A problem which is starting to attract attention is feedback control in quantum transport [79]. In other words, the question is to understand how the current quantum control techniques developed for quantum optical systems can be applied to hybrid systems involving quantum dots, superconducting qubits, opto-mechanical resonators.The existing quantum feedback theory has to be adapted to these new dynamics.

3 Atomic, molecular and chemical physics

3.1 State of the art

Apart from earlier formal statements on quantum optimal control with the Pontryagin maximum principle [40, 20], control within quantum mechanics was first discussed in the context of chemical reactions. It was termed 'coherent control' at the time and conceived as a method to determine the fate of a reaction using laser fields [220, 219, 132]. The basic idea was to employ interference of matter waves to enhance the desired outcome and suppress all others [186, 35]. A way to create the desired matter wave interference is by tailoring laser fields [220, 219, 132]. A reaction is viewed as the following sequence of events—approach of the reactants; formation of a new chemical bond; intermediate dynamics of the generated molecular complex; stabilization into the target products, typically involving the breaking of another chemical bond. Each of these steps can in principle be controlled.

The advent of femtosecond lasers and pulse shaping technology [242, 158] in the 1990s allowed for experimentally testing the idea of controlling chemical reactions. It quickly turned out that the unimolecular bond breaking step is comparatively easy to control, no matter whether one targets dissociation or fragmentation, see Refs. [93, 178, 59, 32, 60, 246, 165, 135] and references therein. Viable control strategies include both weak- and strong-field scenarios. In the first case, a wavepacket is launched, and its ensuing (ro-)vibrational dynamics is exploited. In the strong-field regime, the laser pulse coherently controls the dynamics during the pulse while utilizing the effective modification of the energy levels of atoms [16, 15, 226, 227] or, respectively, the potential energy landscape experienced by the molecules, via the dynamic Stark effect [231, 217, 17].

Weak field control of non-resonant excitation may employ optical interferences to control e.g. the population in a final state [156]. It was proven early in the development of quantum control, that for weak fields in an isolated quantum system phase-only control is impossible for an objective which commutes with the free, or drift, Hamiltonian [36]. A qualitative explanation is that under such conditions there are no interfering pathways leading from the initial to the final stationary states. Experimental evidence has challenged this assertion, claiming demonstration of weak-field phase-only control for an excited state branching ratio [177]. The phenomena was attributed to the influence of the environment. A subsequent study showed that such controllability is solvent dependent [232]. A theoretical demonstration that phase-only control is possible in weak field for an open quantum system soon followed [119]. It is still an open question if weak-field phase-only control is possible for targets that commute with the Hamiltonian in open quantum systems. For example, can population transfer from a ground to the excited surface be phase-controlled for a dye molecule in solution? While this has theoretically been shown to be impossible if the time evolution is Markovian [8], most solvents lead to non-Markovian system dynamics.

In the context of coherent control of a chemical reaction, the bimolecular process of bond formation using femtosecond lasers remained much more elusive [153, 100, 192, 166, 155, 190, 130, 9] than bond breaking. Its coherent control was demonstrated only very recently [142]. Full control of a binary reaction—from its entrance channel of scattering reactants to the targeted products in a selected internal state—is still an open goal. Realizing this dream would create a new type of photochemistry with selective control of yields and branching ratios.

Coherent control of bond formation in the gas phase turned out to be so difficult because it starts from an incoherent thermal ensemble. The laser pulse then needs to pick those scattering pairs which show some correlation in their translational or rotational motion. Averaging over rotations can be avoided by orienting or aligning the molecules using strong laser fields [214]. Adiabatic alignment occurs in-field and is achieved with nanosecond pulses. In contrast, femtosecond laser pulses create non-adiabatic alignment that persists after the pulse is over. This second option where the alignment is produced in field-free conditions is more interesting in view of the applications since the laser pulse does not disturb the molecular system [204, 216].

Spatial averaging in the gas phase implies an integral over the beam profile and blurs coherent effects as the atoms or molecules are exposed to different intensities. Spatial resolution is thus a prerequisite for control [118]. Subwavelength dynamic localization of the laser intensity can be achieved on the nanometre scale [2], lifting the restrictions of conventional optics.

Another way to overcome thermal and spatial averaging is by cooling and trapping the atoms or molecules. Coherent control was suggested as a method to cool internal molecular degrees of freedom [13, 14]. This has been realized experimentally for cooling vibrations of ultracold cesium dimers [208] and rotations of trapped aluminum hydride ions [145]. While restrictions imposed on the cooling efficiency by the molecular structure can be circumvented using optimal control [184], a persistent challenge to cooling internal molecular degrees of freedom are the timescale separation between vibrations and rotations as well as the enormous bandwidths that are required for strong bonds. In order to prepare molecules that are cold in their translational degrees of freedom, molecules are assembled from atoms which are much easier to cool. However, a major problem in creating molecules from atoms is the extreme change in time and length scale. Optimal control was studied to overcome this issue [129]. Initial experiments employed the simple scheme of a chirped pulse to compress two atoms to closer proximity [247]. This can be viewed as a first step in ultracold laser-induced, i.e., photoassociation. Association yielding ultracold molecules in a single internal quantum state has been demonstrated employing magnetic field ramps (magneto-association) followed by STIRAP-type protocols [61, 164]. An experimental challenge still unfulfilled is complete coherent control of ultracold photoassociation.

In addition to making or breaking chemical bonds, coherent control has demonstrated its versatility in studying energy transfer [103] and for spectroscopy, including non-linear and multi-dimensional spectroscopies [refs to be added].

Initially, the coherent control of molecules, be it in the context of chemical reactions or non-linear spectroscopies or energy transfer, considered the dynamics of the nuclear degrees of freedom, using femtosecond laser pulses as the main workhorse. More recently, the focus has shifted to controlling electron dynamics. This is being made possible by the development of advanced x-ray sources which probe the dynamics of electrons within atoms and molecules on attosecond time scales. Their potential for exploring the quantum nature of the nanoworld is unprecedented. For example, using xuv and x-ray light for multidimensional spectroscopy could probe valence excitations locally on different atomic sites in a molecule. This would be invaluable for understanding energy transfer in biological systems and quantum devices. The use of x-ray light sources is currently facing a number of challenges that can be tackled by quantum optimal control. First of all, the large energy of xuv and x-ray pulses results in a high probability of ionization, reflecting the problem of controllability when a continuum of states is involved. This has been addressed in a recent study, where optimal control theory was used to predict experimentally feasible pulses to drive xuv-Raman excitations through the ionization continuum [97].

Another control problem is the creation of the xuv light pulses themselves. In particular, high harmonic generation, where a very strong near-infrared femtosecond laser pulse accelerates an electron in such a way that it emits xuv light, is an ideal candidate for coherent control: Theoretical predictions for optimum driving should be easy to adapt in experiment, given the existing pulse shaping capabilities. The challenge that high harmonic generation poses to optimal control theory is a frequency-domain target [205, 243, 196]. A conclusive answer whether shaping the femtosecond laser pulses can improve high harmonic generation has not yet been provided.

Optimal control theory was first applied to chemical reactions using Krotov's method [221, 209] and gradient ascent [99, 252]. The theory was quickly extended to Liouville space [250, 13] to treat condensed phase situations and cooling. The major experimental constraint in expimerents with shaped femtosecond laser pulses is the fixed bandwidth. This can be accounted for by including frequency filtering in the optimization [92, 199, 139] or by imposing spectral constraints [170, 185]. Optimal control techniques have also been applied with success to molecular alignment and orientation in gas phase [213]. The design of optimal solutions has allowed to reach the best possible degree of alignment and orientation [191] within the experimental constraints such as temperature [138] or collisions [180]. In addition to improving existing control strategies, optimal control has also been used to explore new regimes of alignment dynamics such as planar alignment [107].

3.2 Mid-term prospects: goals and challenges

Thirty years after the conception of reaction control, it is fair to ask whether the idea of coherently controlling a chemical reaction can work at all. In this respect it is important to realize that the basic ingredients for achieving this goal have all been developed. The challenge that remains to be overcome, is their assembly and application to a specific reaction. The dream of coherently controlling a chemical reaction all the way from its entrance channel to the reaction products thus seems to be within reach. It includes the controlled formation (or photoassociation) of a new chemical bond, the controlled dynamics of the intermediate complex, most likely involving a conical intersection, the controlled cleavage of another chemical bond as well as the stabilization of the reaction product. Realistically, demonstration of control over a complete reaction can be expected within the next few years for a sufficiently simple reaction complex, involving only a few atoms.

Another mid-term goal for quantum control is the control of electron dynamics. The capability to control electrons implies Angstrom-scale ultrafast imaging methods which can be realized in the form of laser-induced electron diffraction and high-harmonic spectroscopy. Specific mid-term goals that seem within reach using these tools include the control of subfemtosecond charge migration; the controlled generation of spin-polarized electrons from laser ionization; recognition of the absolute configuration of chiral molecules with shaped laser pulses; and, using high-harmonic spectroscopy of molecules, ultrafast imaging of structure and dynamics on sub-atomic length scales.

3.3 Long-term vision

3.3.1 Synthesis

The ultimate chemical synthetic challenge is to assemble a large chiral molecule from elementary building blocks. Currently such synthesis is carried out in solution where the chemical products are stabilized by entropy generation caused by heat transfer to the environment. The vision would be composed of synthesis by photoassociation via polarization shaped light where the final product is stablized by laser cooling and trapped by light.

3.3.2 Analysis

The vision is a light field tailored to a specific molecule or functional group generating a specific physical outcome such as light emission or ionization. Such a capability will enhance the threshold of detection of a specific hazard or medical application. Combined with spatial super-resolution, the analytic methods will be employed in molecular based microscopy.

4 Magnetic resonance

The optimal control of spin dynamics is at the heart of well established magnetic resonance technologies and of emerging new fields of quantum technologies. Nuclear magnetic resonance spectroscopy (NMR) [82, 1], electron spin resonance spectroscopy (ESR) and magnetic resonance imaging (MRI) are based on the cotrol of nuclear spins and electron spins with the help of time-dependent electromagnetic fields. In fact, magnetic resonance is one of the most impressive success stories of quantum control and technology.

The mathematical description of nuclear or electron spins and their dynamics is essentially identical to the canonical description of abstract quantum bits (qubits). In terms of control of spins or qubits, NMR had a long head start compared to other quantum technologies as the community has actively explored and developed control methods for more than 60 years. This was driven mainly by very concrete and powerful applications in physics, chemistry, biochemistry, biology and medicine and the interdisciplinary impact of quantum-control enabled magnetic resonance is impressively reflected by the Nobel prizes in Physics (Felix Bloch, Edward Purcell, 1952), Chemistry (Richard Ernst, 1991; Kurt Wüthrich; 2002), and Medicine (Paul Lauterbur, Peter Mansfield, 2003) [81, 248, 152]. Today, NMR is arguably the most important molecular structure determination tool in chemistry, ESR is an essential technique in radical reaction chemistry, catalysis, electrochemistry and photosynthesis research and MRI is one of the most informative and frequently used modalities in medical diagnostics. The huge range of practical applications has generated a new multi-billion dollar instrument manufacturing industry (Bruker, Siemens, Phillips, General Electric, JEOL, etc.). This in turn has resulted in the continuous development of more and more sophisticated instruments with superb flexibility in terms of the available control schemes: For example, arbitrary waveform generators and linear amplifies are already standard NMR equipment since more than three decades (and have more recently also become commercially available with sub-nanosecond time resolution for ESR applications). With their help, very complex pulse shapes can routinely be implemented with high fidelity. The excellent agreement between theory and experiments (as a result of the highly accurate theoretical description of the physics of coupled spins and the availability of very reliable hardware to implement virtually arbitrary pulse sequences and shapes) also has made NMR an attractive testing ground for the experimental demonstration of new control approaches for finite-dimensional quantum systems in general. Concepts of quantum control and sophisticated quantum-control design tools developed in the field of NMR have found many applications in other fields, such as in quantum information processing, optics (photon echos), neutron scattering, in the control of nano devices based on quantum dots, artificial atoms etc.

Emerging fields of magnetic resonance are hyperpolarization methods for bulk NMR and the control and measurement of individual spins or spin systems, e.g. of NV centers in diamond with many potential applications in sensing and quantum information processing.

Hyperpolarization techniques (also known as spin cooling), can generate highly polarized non-thermal spin states. Hence, the relatively low sensitivity of NMR (due to the small magnetic moments of nuclear spins and the resulting weak thermal polarization) can be overcome by using a variety of approaches. In particular, there are two different methods that have recently become increasingly popular in practical applications. The first one is based on transfer of the much higher polarization from unpaired electrons onto the nuclear spin ensemble in a process that is called dynamic nuclear polarization (DNP). The second method involves the use of parahydrogen and a transfer of its highly populated singlet spin state onto nuclear spins in receptor molecules. Both methods have been known already for many years but only recently significant progress has been made in terms of a full quantum description of the underpinning spin physics and the optimization of the required experimental hardware.

The detection and control of individual nuclear spins close to Nitrogen vacancy (NV) centers in diamond is a premier example of new area of optimal control of individual spin systems. In these experiments, single nuclear-spin detection efficiency is achieved by an efficient readout based on couplings of the nuclear spins to electron spin states and their efficient readout using optical techniques.

4.1 State of the art

Geometric and numerical tools from optimal control theory have not only provided pulse sequences of unprecedented quality and capabilities, but also new analytical insight and a deeper understanding both of the mode of action of optimal pulses. Numerically optimized pulses can often be interpreted as robust variants of optimal trajectories. These can be understood based on geometrical concepts and useful tools have been developed to analyze complex pulses [131] and the resulting dynamics in coupled spin systems [136].

On the industrial development side, optimal control theory research in magnetic resonance encouraged the development of new hardware – the recent release by Bruker of a sub-nanosecond arbitrary waveform generator (AWG) has made it possible to start exploring the physical limits of the ESR experiments [211].

4.2 Mid-term vision

An important goal is to make optimal control algorithms easier to use, generally applicable and to further increase their speed. Depending on the applications, very different convergence rates were encountered and a systematic characterization of optimal control landscapes is still missing. With more efficient numerical optimizations methods, optimal control theory will make it possible to design problem-, sample- and patient-specific pulse sequences interactively. The fast reoptimization of pulse sequences or of sequence elements, e.g. in response to the presence of magnetic susceptibility jumps, will significantly improve their performance. In addition to MRI applications, this could be important in production or process monitoring by NMR, where the pulse sequence should be able to adapt to the sample in the same way as shim currents currently do – one example is oil well logging (susceptibility and tuning variations), another one is metabolomics (salinity and chemical composition variations). Optimal control methods are expected to reduce the time which is required to determine structural and dynamical information of biomolecules (e.g. proteins). In this field, coping with large coupled spin networks, especially in the presence of relaxation is computationally hard and further improved numerical/analytical approaches are highly desirable. Characterizing the experimental imperfections and fine-tuning the spectrometer and setting up the experiment is time-consuming and efficient closed loop feedback-based automatic procedures have to be developed, implemented and integrated with the adaptive design of pulse sequences to automate this process. One possible way to better optimization algorithms could be the combined use of numerical algorithms and geometric optimal control methods.

It the field of medical imaging it is expected that optimal control methods will lead to more sensitive and more efficient pulse sequences, such that a patient has to spend less time in a scanner for an examination. This may be achievable by using multi-band excitation techniques and optimized image acquisition based on multiple transmit and receive channels. Optimal control methods are also expected to help in the extension of the clinical applicability of ultra-high-field scanners and to provide in-vivo spectrosopy with improved diagnostic value. Apart from better localized excitation, progress in the field of magnetization preparation (e.g. reducing the B_0 sensitivity of fat saturation) might be achievable. Improved saturation pulses are expected to be useful for many different task, from chemical shift imaging (CSI) and chemical exchange saturation transfer (CEST) imaging to single voxel spectroscopy. Also methods for improved quantification accuracy and biomarker imaging are highly desirable.

In the field of hyperpolarization, further hardware advances are expected, in particular in generating and modulating high frequency microwave fields and it is envisaged that optimal control methods will play an important role in the development of more sophisticated experimental schemes to transfer the electron polarization to surrounding nuclear spins. Ideas in this respect have already been published and discussed including the manipulation of the nuclear spins using the anisotropy of the hyperfine interaction [122, 104] and the exploitation of repeated generation of dipolar spin order to enhance polarization transport by spin diffusion [65]. Since more sophisticated catalysts are being developed for parahydrogenation reactions, it is also anticipated that pulse shapes derived from optimal control principles will be more frequently used to maximize the achievable polarization and to mediate polarization transfer to specific molecular site.

4.3 Long-term vision

Very important theoretical and practical aspects of optimal control research in magnetic resonance are the limits of quantum dynamics and optimization theory. On the one hand, the questions of quantum state reachability in dissipative systems remain largely unresolved. On the other, many practical usage scenarios are time-constrained, and a more general understanding of the best possible performance in a give amount of time is highly desirable.

A very desirable outcome of the continued progress in optimal control technology could be that, for a given level of performance, the use of optimal control sequences could significantly reduce the instrument costs as well as costs of sample preparation and purification. Sophisticated shim coils, frequency locks, complicated combinations of susceptibility matched materials in NMR probes and other expensive arrangements had originally been introduced to maximize spectral resolution and selectivity. If both could be achieved by tailored pulse sequences under less than ideal conditions, the complexity could be transferred from the instrument design to the mathematical optimization procedure. The concomitant reduction of hardware cost could result in more affordable instruments, e.g. for MRI examinations. Also, the integration of control design with image reconstruction and spectral calculation in NMR and with instrument design could result in better performance. The combination of open loop and feedback strategies may result in fast and fully automated tune-up procedures, which would further reduce the required experimental time.

A long-term vision/dream of magnetic resonance techniques is the detection of the nuclear spins of s single molecule (e.g. of a protein). The ability to image the shape of individual molecule similar to the way we can image humans today would revolutionize structural biology and the rational and efficient development drugs for diseases. The recent developments in sensing sensitivity of NV centers in diamonds may provide a potential road to this goal, as well as completely new application areas e.g. in medical diagnostics.

5 Quantum information and communication

5.1 State of the art

Quantum technologies (see, e.g., [74]) exploit quantum coherence and entanglement as essential elements of quantum physics. Applications include high-precision measurements and sensing, which would reach unprecedented sensitivity, the simulation of physical and biological systems, which would be impossible to study otherwise, and quantum information processing, which would allow to solve computationally hard problems. Successful implementation of quantum technologies faces the challenge to preserve the relevant nonclassical features at the level of device operation. More specifically, each task of the device operation needs to be carried out with sufficient accuracy, despite imperfections and potentially detrimental effects of the surroundings. Quantum optimal control not only provides toolboxes that allow for identifying the performance limits for a given device implementation, it also provides the protocols for realizing device operation within those limits.

Prominent tasks include the preparation of useful quantum states as well as implementation of quantum operations. The power of the quantum optimal control approach for implementing these tasks has very recently been demonstrated in a number of impressive experiments. For example, nonclassical motional states of a Bose-Einstein condensate were prepared with optimized control sequences for wavepacket interferometry [233], and the loading of an ultracold atomic gas into an optical lattice was improved [188]. With respect to quantum operations, quantum optimal control allowed for error resistant single-qubit gates with trapped ions [224] and for single qubit gates without the need for invoking the rotating wave approximation in nitrogen vacancy (NV) centers in diamond [197]. For the latter platform, optimal control is also at the basis of a spectroscopy protocol allowing to image nanoscale magnetic fields [102]. In quantum processor candidates based on superconducting circuits, leakage to non-computational states in the most common type of qubit, the transmon [161, 49, 148], was avoided and frequency crowding was accomodated [203, 235] thanks to optimal control results. Closed-loop optimal control [120, 75] enabled finetuning of gates that were determined manually, allowing them to reach consistent record fidelities within this platform. In view of scaling up control, the design and implementation of unitary maps have recently been demonstrated in a 16-dimensional Hilbert space, spanned by the electron and nuclear spins of individual cesium atoms [10].

The use of control methods in a broader sense has allowed for further significant experimental achievements, such as to improve of the coherence of a qubit, realized by the electron spin in an NV center, using dynamical decoupling [41]. A famous further example of high-end control techniques is the Paris experiment of stabilizing a quantum state with predefined photon number via real-time closed-loop feedback [195], which required to include the noise back-action of controls onto the system by way of stochastic differential calculus.

These experimental achievements were preceded by a large number of theoretical predictions on how optimal control may improve or enable quantum state preparation, operation and readout. State preparation protocols include transport of atoms [62] and ions [206, 86] as well as transport in a spin chain [43, 163], photon storage [94], preparation of squeezed states [98], cluster-states [83], nonclassical states in a cavity [187] or in spin chains [239, 240], as well as preparation of a quantum register [72] and many-body entangled states [45]—to name just a few.

Likewise, optimal control helped to implement high-fidelity quantum gates such as two-qubit gates with neutral atoms in dipole traps [42, 73], on atom chips [228], or with Rydberg atoms [54, 89], two-qubit gates between ions [176], between an ion and an atom [68], error-correcting qubit gates of electron and nuclear spins within single NV centers [237], or entangling gates between distant NV centers [70]. For superconducting qubits, see also the review in [50], two-qubit gates were optimized, starting from Cooper pair boxes [159, 212, 111, 55] to modern transmon-based schemes [77, 91]. In these optimizations, special attention was paid to robustness against noise [159, 181] which can even be used as a tool for control [182]. Also, readout has been addressed [76]. In order to adapt to the strong filtering of control lines in superconducting qubits, transfer functions had to be taken into account [109, 160] and experimental fluctuations and noise were accomodated [239, 91]. Fidelity limits on two-qubit gates due to decoherence were studied for Markovian [201, 90, 91] as well as non-Markovian [84] time evolutions.

In view of quantum computation, it has been suggested how to retain universality in spite of limited local control [239, 39], or by using environmental degrees of freedom [182]. The Jones polynomial, i.e., a central invariant in knot theory, can be evaluated, using an NMR spin ensemble at ambient tempreature, in an algorithm equivalent to deterministic quantum computing with a single pure qubit [171, 154, 200].

In order to obtain these results, the quantum optimal control methodology had to be adapted to the requirements of Quantum Information and Communication Technologies. Optimization algorithms had to be derived for specific quantum gates [168, 169, 222], dissipative evolution as seen in the reduced system dynamics [115, 181, 201, 90], or exploiting invariants in system-bath models [96], optimization up to local equivalence classes [162], which can also be used for arbitrary perfect entanglers [241, 88] or optimizing for many-body entanglement [174]. Moreover, control techniques were adapted to non-linear dynamics as found in a BEC [207, 106] and to general dynamics, functionals and couplings to be controlled [183]. Many-body systems that are too large for gradient-assisted types of algorithms can be optimized numerically in CRAB [72, 44] by interfacing t-DMRG with parameter optimization (e.g., via the SIMPLEX algorithm). Other techniques specifically cover robustness with respect to experimental fluctuations or noise [239, 91] or filters in experimental implementation of controls [109].

5.2 Mid-term prospects: goals and challenges

The field of quantum technologies has matured to the point that quantum enhancement is explored beyond quantum computation only. Devices such as quantum simulators or quantum sensors are currently under active development. Control methods will be crucial to operate these devices reliably and accurately. This involves the device preparation, or reset, the execution of the desired time evolution, and the readout of the result. These tasks set the agenda for the next few years.

More specifically, central mid-term milestones include the robust implementation of gates in a multi-qubit architecture, finding solutions to readout and fast reset limitations, automatization of key tasks of surface code error correction and optimal as well as robust generation of multi-particle entangled states for a variety of QICT platforms. All of these will require decoherence control.

Considering superconducting qubits as one example platform, a main challenge for optimal quantum control is to reach convergence of numerical optimal control and experimentation. To date, either optimal control is used for computer-aided discovery of analytical schemes that can be remodeled in an experiment or the superconducting quantum processor itself is employed to calibrate gates. In order to better combine numerical optimal control and experimentation, the modeling of superconducting qubits as well as errors and other non-idealities of the system, in particular for open systems, needs to be improved and the robustness of pulses enhanced. Also, optimized pulses should initially be reduced to few parameters before more complicated and effective solutions can be pursued. On the other hand, pulse shaping platforms need improvement. Applications of quantum control in superconducting qubits should follow the current European thrust towards analog and digital quantum simulation and lead to the preparation of entangled ground states, fast and accurate quantum gates and tools for quantum machines. For some instances, compatibility with quantum error correction is desired.

For trapped ions, it seems realistic to combine quantum gates with ion transport in segmented traps, using optimal control.

Tommaso, can you please add midterm goals for other platforms such as (Rydberg) atoms, NV centers? (we didn't get any specific input there)

In general, control techniques are expected to contribute to decoupling and dissipative state-engineering [38, 234, 133], for instance in view of enhancing the lifetime of quantum memories. In order to improve the lifetime of a quantum register, control can also be used to implement error-correcting gates and circuits [237]. Moreover, while quantum compilation, i.e., the translation of a unitary gate into the machine language of pulses and evolutions, can readily be done via optimal control up to some 10 qubits [202], a scalable assembler of elementary gates (up to 10 qubits) into many qubits is an open problem that may benefit from tensor-contraction techniques like DMRG and PEPS.

Both numerical optimal control and closed-loop control are expected to be useful for tackling these goals. Numerical optimal control has the advantage of versatility, whereas closed-loop control can easily be tuned to specific tasks such as determining parameter uncertainties. A hybridization of both approaches is conceivable as well. The main difficulties that need to be overcome to reach the above mentioned mid-term milestones are a sufficiently accurate modelling of complex quantum dynamics to build control on top, integration of tomography and system identification with optimal control, efficient ways to take into account experimental constraints and uncertainties, and bridging the gap between the quantum control community and the communities of the respective QICT platforms.

5.3 Long-term vision

Several current QICT platforms show a strong scaling potential. Thus in the long term, control schemes need to be made scalable. This represents a severe challenge, but meeting this challenge will make quantum control a basic building block of every quantum technology and ensure, at the same time, their proper functioning in a world that is only partially quantum.

Take the example for superconducting qubits where fabrication is a key task that could and should be improved by control techniques. The controlled adjustment of fabrication parameters should be simple, and the qubits should to a certain extent be robust to the influences of the rest of the architecture they are placed in. Independent of a specific platform, error correction at large, for instance by toric codes [85, 12], is one of the strategic long-term goals that is expected to benefit from control techniques given recent advances by randomized benchmarking [120]. To this end, system-identification protocols matched with optimal control modules will be of importance. A pioneering step in this direction was made by the ADHOC technique [75] that combines open-loop control as a first step with closed-loop feedback learning (with implicit parameter identification). Moreover, taking quantum control algorithms to match with tensor-contraction techniques in order to address quantum-many body systems (where first steps have been made by CRAB [72, 44]) will be a challenge paving the way to more accurate handling of experimental quantum simulation setups.

Tommaso, any long-terms goals to add that should become reachable thanks to optimal control?

In short, quantum control will be the means to get the most performance out of an imperfect system and combine challenging physics at the few-qubit level with engineering at the multi-qubit level. This should aim for example at enabling quantum simulations that are impossible on classical computers. In addition, the realization of the following long-term goals, using optimal control techniques, seems challenging yet conceivable: demonstrating the practical usefulness of engineered quantum states, for example in quantum metrology; implementing reliable strategies for the control of mesoscopic systems; exploring the dynamics of quantum many-body systems beyond equilibrium; and understanding the microscopic origin of thermodynamic laws.

6 Prospects for applications and commercial exploitation

Quantum-control enabled technologies have potential for truly revolutionary innovation. More sophisticated quantum control techniques are making current technologies more powerful and also help to create novel technologies, e.g. in sensing with super sensitive magnetic detectors, microscopic temperature measurement devices, molecular imaging etc. Better control of quantum systems has the potential to significantly reduce instrument costs, turning perhaps million dollar NMR spectrometers into small and portable devices with many new fields of applications.

As visible above, quantum optimal control applications broadly fall into two classes: Applications to genuine *quantum technologies* and applications in chemistry-related areas such as spin resonance.

The magnetic resonance industry as we know it today would not exist without quantum control and novel optimal control strategies have already been implemented in commercial NMR spectrometers and implementations of optimal control sequences in MRI are being pursued. Clearly, with more and more improved sequences being developed, this trend is expected to continue.

On the other end of the spectrum, there is presently an emerging industrial effort in Quantum Computing lead by IBM, Google, and Microsoft. The first two companies have invested into the development superconducting qubits and both of them use optimal control techniques [120, 55]. This is not coincidence - optimal control can have impact in systems that have reached some technical maturity in research laboratories, which is the point at which industry gets interested. Further industrial perspectives will be linked to the further development of a quantum technologies industry. Early convergence could happen in quantum sensing, which takes sensing ideas similar to those appearing in spin resonance and combines them with ideas from quantum technologies.

7 Conclusions

Quantum control is a key facilitator for spectroscopy, imaging, AMO physics and emerging quantum technologies for computation, simulation, metrology, sensing and communication. For all these application, it is crucial to reach the required precision given experimental limits on control amplitudes, power, timing, accuracy of instruments as well as the ever-present interaction with the environment. Optimal control theory provides a framework to identify which quantum tasks can be accomplished with what precision in the presence of decoherence and experimental imperfections and limitations.

Quantum control systems theory will require the integration of control aspects at many different levels. Future quantum technologies will rely on integrated architectures of hybrid quantum systems with e.g. nuclear spins for long-term storage, quantum-nanomechanical devices for sensing and photons for the communication of quantum states. This will require also the integration of quantum mechanics in engineering education and vice versa. It will be necessary to establish strong links of quantum control experts to quantum engineering and to the manufacturing of quantum devices.

Due to its interdisciplinary nature with applications in many diverse fields, future advances in the optimal control of quantum systems will require the combined effort of people with expertise in a wide range of research fields. Only the close link of basic research, development and application will open scientifically and economically rewarding perspectives and will foster the innovation potential of emerging quantum technologies in an optimal way. The Virtual Facility for Quantum Control (VI-QC) under the umbrella of the EU Coordinated Action for Quantum Technologies in Europe (QUTE-EUROPE) marks a important step in this direction. The primary goal of the VF-QC is to provide a common structure for the growing quantum control community in Europe, for the promotion of quantum control and to provide expertise to other scientific communities, to policy makers and the general public. Establishing common terminology, common standards and common visions are crucial requisites to maximize the beneficial impact of optimal quantum control methods on current and future technology, economics and society.

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